Example Midterm Problems
"Fixed Income Analysis"

Problem 1

The following parameters $b_0 = -0.326, b_1 = 0.34, b_2 = 0.56$, and $\lambda = 0.03$ determine the shape of a fitted Nelson Siegel term structure function.

a) Compute the prices of zero coupon bonds with maturity in 6, 12, 18 and 24 months from now.

b) Compute the value of a 5% semi annual coupon bond with exactly two years to expiration.

c) The two year, 5% coupon bond is trading at 102. Is there an arbitrage? If so detail what to buy and sell in order to produce the arbitrage profit. Assume that you can trade the zero coupon bonds in a).

d) Answer question c) again assuming that you cannot trade the 1.5 year zero.

e) What is the accrued interest on the 2 year, 5%?

f) If you were to quote a price on the 2 year as a dealer, what would you quote? Round of to the nearest 32d.

g) You observe that two 4% coupon bonds with 1.5 and 1 year maturities are trading at 104.13 and 102.24 respectively. Suggest a barbell trade. Assume you can trade the 2.5 year, 5% at 105.8.

h) Compute the Macauley durations for the three bonds in g).

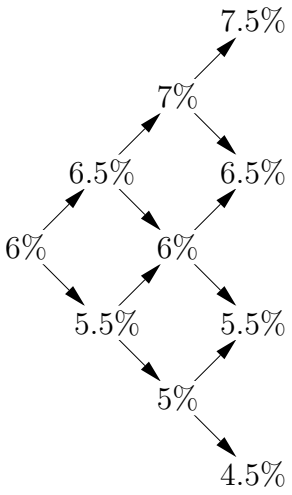
i) Compute the dollar durations using the relationship between Macauley and dollar durations.

j) Compute the dollar duration of the portfolio in g)

k) Suppose market prices change. You re-fit the Nelson-Siegel model and find $b_0 = -0.32$. Other parameters are as before. Does your hedge work?

Problem 2

Take the following interest rate tree with six months long branches:



a) Find the price of a 5%, 2 year coupon bond at each node in the tree.

b) Find the value of an option to buy the bond in a) at par with maturity 0.5, 1, and 1.5 years.

c) With stocks, call options increase uniformly in value with the maturity of the option. Is this true for bonds?

d) Find the price of a constant maturity treasury swap that pays

$$\frac{1000}{2}(r_t - 5\%)$$

e) You are working for an investment banks (remember those?) which have asked you to price a special derivative contract that pays $ 100 if the spot rate is between 5 and 6.6% in a year and a half from now. What should you charge the customer?