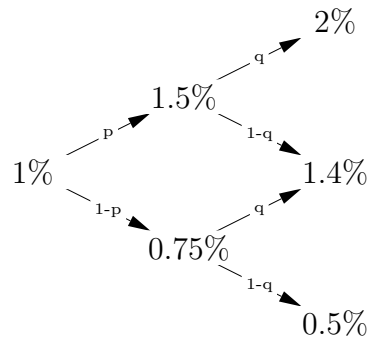


# Example Problems Final Exam

## "Fixed Income Analysis"

**Problem 1** Consider the tree:



The prices of 1.5, 1, and .5 year zeros are 98.296, 98.946 and 99.5025 respectively.

- Find the risk neutral probabilities  $p$  and  $q$ .
- Price a 6 month option on the 1 year zero with strike  $K = 99.50$ .
- Price a 1.5 yr bond that pays a semi annual coupon of 0.85895.

d) Consider a plain vanilla swap with 2 year maturity. Here the fixed and floating rate payments occur every six months. What is the swap rate?

e) Answer question c) again assuming that the reference rate is LIBOR which we assume is always 25 BP above the spot rate.

f) Suppose we are receiving fixed and paying floating. What would we need to do to get out of the swap after 6 months?

g) A 6 month swaption is trading at 22 cents. Recommend a trade.

h) A caplet pays

$$10,000,000 \times \max(R(t) - 1\%, 0) \frac{180}{360}$$

Find the value of a 6 and 12 month caplets and a 12 month cap.

i) Find the price of a 1% semi annual bond with 2 year maturity at each node in the tree.

j) Find the price of a 1.5 year American option (strike=100) on the 1% coupon bond in i).

k) Find the value of a 1% coupon bond that is callable anytime. Write down the value of the callable bond in the tree.

l) Find the value of an option on the callable bond with 6 month maturity and a strike of 99.90.

**Problem 2**

Note: This is a problem that involves non-recombining trees.

A special futures contract is trading on the average of the spot rate over a 12 month period. Specifically, in 12 months this contract pays

$$1000\bar{R}$$

where  $\bar{R}$  is the average spot rate (in percent) over the last 12 months. For example, a 1.5 year futures will pay

$$1000 \times \left(\frac{1}{2}1.5 + \frac{1}{2}2\right) = 1750.$$

if rates go up twice over the first 12 months.

The futures is marked to market every 6 months (no cash exchanges hands at time 0).

a) Find the futures prices at each node in the tree.

b) Show the cash flows from the buyer of the futures at each node in the tree.