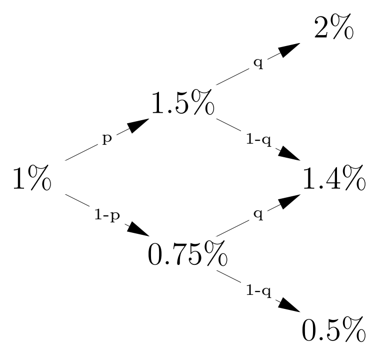
Example Problems Final Exam
"Fixed Income Analysis"

Problem 1 Consider the tree:



The prices of 1.5, 1, and .5 year zeros are 98.296, 98.946 and 99.5025 respectively.

a) Find the risk neutral probabilities p and q.
$p = q = 0.5$.

b) Price a 6 month option on the 1 year zero with strike $K = 99.50$.
0.0629

c) Price a 1.5 yr bond that pays a semi annual coupon of 0.85895.

99.5705

d) Consider a plain vanilla swap with 2 year maturity. Here the fixed and floating rate payments occur every six months. What is the swap rate?

1.1484

e) Answer question c) again assuming that the reference rate is LIBOR which we assume is always 25 BP above the spot rate.

1.1484+.25=1.3984

f) Suppose we are receiving fixed and paying floating. What would we need to do to get out of the swap after 6 months?

We are essentially long a regular bond with fixed coupon. We must compensate an amount equal to the difference between these numbers and par. The value falls when interest rates go up, and the fixed leg is worth 99.554. We must compensate the fixed rate payer 0.446 = 100-99.554. In the down-state we similarly receive 0.297 if we opt out of the swap.

g) A 6 month swaption is trading at 22 cents. Recommend a trade.

The theoretical value is 0.1479 and we can arbitrage the difference by buying a hedging portfolio consisting of two differently dated bonds.

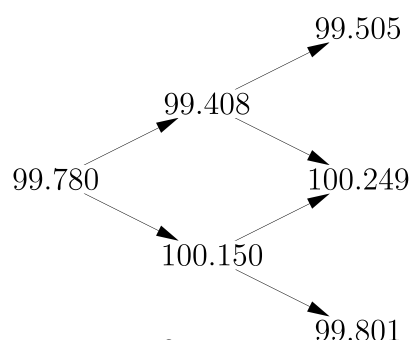
h) A caplet pays

$$10,000,000 \times \max(R(t) - 1\%, 0)\frac{180}{360}$$

Find the value of a 6 and 12 month caplets and a 12 month cap.

The 6 and 12 are worth 1244 and 2224 and the cap is the sum which is 3468.

i) Find the price of a 1% semi annual bond with 2 year maturity at each node in the tree.



j) Find the price of a 1.5 year American option (strike=100) on the 1% coupon bond in i).
0.0745

k) Find the value of a 1% coupon bond that is callable anytime. Write down the value of the callable bond in the tree.

It is 99.705 at time 0. It is 100 on the lower branches and identical to the non-callable elsewhere.

l) Find the value of an option on the callable bond with 6 month maturity and a strike of 99.90.
0.0498

Problem 2

Note: This is a problem that involves non-recombining trees.

A special futures contract is trading on the average of the spot rate over a 12 month period. Specifically, in 12 months this contract pays

$$1000\bar{R}$$

where $\bar{R}$ is the average spot rate (in percent) over the last 12 months. For example, a 1.5 year futures will pay

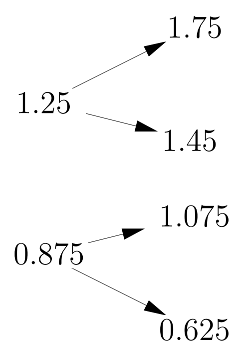
$$1000 \times (\frac{1}{2}1.5 + \frac{1}{2}2) = 1750.$$

if rates go up twice over the first 12 months.

The futures is marked to market every 6 months (no cash exchanges hands at time 0).

a) Find the futures prices at each node in the tree.

We get the following non-recombining tree for $\bar{R}$:



Consider first what must be the fair futures price for in the up state at time 0.5. There are two possible future states 1.75 and 1.45. If we enter a futures contract at this time and state, we pay nothing, but settle the contract at time 1. The expected payoff is $(1.75{+}1.45)/2 = 1.6$. The price of a futures with 12 month maturity is therefore \$ 1.6 in the up-state at time 0.5.

Similarly we find that the price is 0.85 in the down state at time 0.5

Finally, we find the price at time 0 as $(0.85 + 1.6)/2 = 1.225$.

[note: we could have concluded this by just averaging the possible values of $\bar{R}$ at time 1.]

b) Show the cash flows from the buyer of the futures at each node in the tree.

The cash flow will be the difference between the old and the new futures price (marking to market).

At time .5 in the up-state:

$$1.6 - 1.225 = 0.375$$

and in the down-state

$$0.85 - 1.225 = -0.375$$

At time 1 in the up-up state:

$$1.75 - 1.6 = 0.25$$

in the up-down state:

$$1.45 - 1.6 = -.25$$

In the down-up state:

$$1.075 - 0.85 = 0.225$$

and in the down down state

$$.625 - 0.85 = -0.225$$