

Regression Based Hedging

Bjørn Eraker

Wisconsin School of Business

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When Duration Neutrality Fails

Last episode of "Fixed Income" :

Hero trader loses money on dollar duration neutral portfolio after non-parallel shift in yield curve.

What happened? The 2010 maturity moved 20 basis points while the 2020 maturity moved 10. Hero lost \$1.18.

If we only knew that a typical interest rate change would imply a 20/10 basis point change in the two bond yields, we would have been able to adjust our hedge accordingly....

The example was: One short 8.75 of 8/2020 worth 142.931 at a ytm of 4.218 hedged by long 7.4532 of 2.125 of 4/30/2010. This bond has price 101.006 at a ytm of 2.025.

The dollar durations (Bloomberg) were 11.627 and 1.56 respectively

Given 10/20 basis points drop in interest rates, we had a total P&L of \$-1.18

Suppose we knew that the yield on the 2010 was likely to change by an amount twice that of the 2020 (just assume).

In other words: the price of the 2010 maturity changes by a factor of 2 times that of the 2020 given the "event".

Lets try to adjust the hedge ratio by 2...

Now we long 14.9 bonds of 2010 for each short 2020.

The P&L for the 20/10 basis point drop is now -0.011...

Our strategy is successful but only because we were able to predict that an interest change would lead to a move in the short maturity twice that of the long maturity.

We cannot know exactly how much the yields will change in relation to each other, however, we can measure a typical change....

Measuring Expected Yield Changes by Regression

We are interested in the question: How much does the yield on bond A typically change when the yield on bond B changes by one basis point?

To answer this consider the regression

$$\Delta y_A = \alpha + \beta_{A,B} \Delta y_B + \epsilon \quad (1)$$

and let's hope that $\alpha \approx 0$. Then the answer to our question is that the price of bond A changes by $\beta_{A,B}$ basis points when the yield on bond B changes by one basis point. On average.

We deduce that in general our hedge needs to satisfy

$$0 = F_A \times \beta_{A,B} \frac{DV01_A}{100} + F_B \times \frac{DV01_B}{100}.$$

The optimal hedge is

$$F_A = -F_B \frac{DV01_B}{\beta_{A,B} DV01_A} \quad (2)$$

Note that we can instead run the regression

$$\Delta y_B = \alpha^* + \beta_{B,A} \Delta y_A + \epsilon \quad (3)$$

In which case the hedge ratio is

$$F_A = -F_B \beta_{B,A} \frac{DV01_B}{DV01_A} \quad (4)$$

Note: If the intercepts in these regressions are zero, then

$$\beta_{A,B} = \frac{1}{\beta_{B,A}}$$

and it does not matter which way you run the regression.

Lets try....

We collected monthly data on prices of Feb 11, 21 and 31:

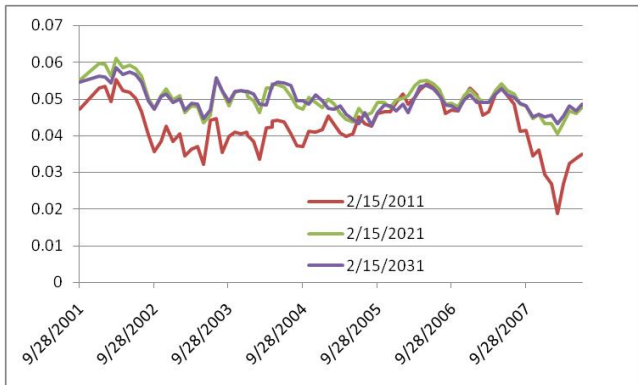


Figure: Yields on 2011, 2021, and 2031 Feb maturities.

We run two different regressions. First, lets regress 2011 on 2021:

We get

$$\beta_{11,21} = 1.087$$

with an $R^2 = 0.58$.

Next, lets regress 2021 on 2011. We get We get

$$\beta_{21,11} = 0.533$$

with an $R^2 = 0.58$.

Now look at the market on 2/28/2002. At this time, the DV01 of 2011 was 0.072 and 0.1979 for the 2021 issue.

Suppose we are short one bond of 2021, so $F_{21} = -1$. ($B =$ the 2021 issue).

Using the 21 on 11 regression we get

$$F_{11} = 0.533 \frac{0.1979}{0.072} = 1.465 \quad (5)$$

Note that we adjust our hedge by a full 53.3% relative to the pure \$ duration neutral hedge which calls for $0.1979/0.072 = 2.75$.

If we use the regression of 11 on 21 we get

$$F_{21} = \frac{1}{1.087} \frac{0.1979}{0.072} = 2.53 \quad (6)$$

A relatively modest adjustment.

We want to see how our hedge works. Empirically, we can do this by *backtesting* using historical data.

The excel spreadsheet "regressionHedge.xls" does this.

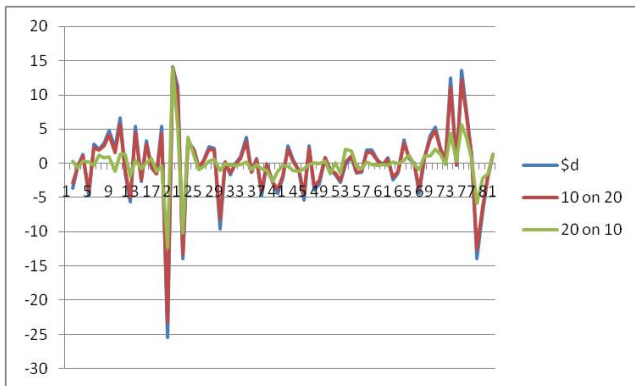


Figure: Hedging errors from hedging a short position in 2021 with the 2011. The blue line gives is just dollar duration, the red adjusting with the 11 on 21, and the green adjusts with the beta of 21 on 11.

It is fairly evident that the green does better. The standard deviations of the hedging errors are 5.5, 4.98, and 2.88.

Conclusion: Our small empirical test suggests that we are better off running a regression of the long dated bond on the short, and adjusting the hedge accordingly.

Two Variable Regressions

Consider a market maker who is long a 20 year.

It is almost certainly that he would do a better job predicting the changes in his long position using both a 10 year and a 30 year bond.

To this end, consider the regression

$$\Delta y_{20} = \alpha + \beta_{10} \Delta y_{10} + \beta_{20} \Delta y_{30} + \epsilon$$

Consider taking positions

$$F_{10} = -F_{20} \frac{1}{\beta_{10}} \frac{DV01_{20}}{DV01_{10}}$$

and

$$F_{30} = -F_{20} \frac{1}{\beta_{30}} \frac{DV01_{20}}{DV01_{30}}$$

in the 10 and 30 year issues.

We can show that

$$P \& L = F_{20} DV01_{20} [\beta_{10} \Delta y_{10} + \beta_{30} \Delta y_{30} - \Delta y_{20}]$$

so

$$E(P \& L) = F_{20} DV01_{20} \alpha \approx 0$$

if $\alpha \approx 0$ (in expectation).

Let's see how it works...

We find

$$\beta_{11} = 0.3604$$

and

$$\beta_{31} = 0.5154$$

with an R^2 of .72.

The total hedging error is now only 2.3.

The reduction in hedging errors come from two sources:

- The use of two instruments. Two is better than 1...
- The use of a better instrument for hedging: the 31 maturity. This bond has much higher correlation with the 21 than does the 11.

Lets just forget about duration measures. How about we try to figure out how a price change in the three bonds will co-vary?

So lets run the regression

$$\Delta P_{21} = \alpha + \beta_{11} \Delta P_{11} + \beta_{31} \Delta P_{31} + \epsilon$$

We now need to hold $F_{11} = -\beta_{11}$ and $F_{31} = -\beta_{31}$ to minimize the hedging errors.

I computed $\beta_{11} = 0.9061$ and $\beta_{31} = 0.4540$.

This gives a pricing error (standard deviation) of 2.03.

I win