

Swaps

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Interest Rate Swaps

An interest rate swap is an agreement between two parties to exchange fixed for floating rate interest rate payments.

- The floating rate leg is typically pegged to LIBOR
- The fixed rate leg is determined at initiation

Fixed rate is determined in such a way that the market is indifferent between holding the floating/fixed rate leg of the contract. Thus, entering into the swap arrangement is *costless* at time 0.

Example

Table: 18.4 Two year cash flows on a 10 year, 5.688% fixed rate swap

date	libor	date	days	floating receipts	30/360	fixed payments
11/26/01	2.156	11/28/01	88			
02/26/02	2.000	02/28/02	92	550,883	90	
05/26/02	1.900	05/28/02	89	494,444	90	2,844,000
08/26/02	2.000	08/28/02	92	485,556	90	
11/26/02	2.100	11/28/02	93	516,667	91	2,859,800
02/26/03	2.200	02/28/03	91	530,833	89	
05/28/03	2.300	05/28/03	89	543,889	90	2,828,200
08/26/03	2.400	08/28/03	92	587,778	90	
		11/28/03	92	613,333	90	2,844,000

For example, the cash flow on 05/28/02 is based on the 02/26/02 LIBOR of 2%. With 100M notional amount, the fixed receives

$$100,000,000 \times \frac{2\% \times 89}{360} = 494,444$$

from the floating payer and pays

$$100,000,000 \times \frac{5.688\% \times 180}{360} = \$2,844,000$$

on 05/28/02. Here, the fixed rate payer receives some 500K every quarter and pays about 2.8 M every 6M for a combined average loss of almost 1.8M every 6m.

Valuation of swaps

Principle: Arrange payments such that the floating and fixed sides are indifferent.

Lets think of the swap as being a long + short position in two bonds:

- One fixed rate with semi-annual coupons
- One floating rate note with quarterly coupons where the floating rate pegged to the previous period Libor
- Make sure the value of both is par
- If so, both the principal and the par values cancel in the long-short portfolio

Digression: Par Yield Curves

A par yield curve represents the ytm (or coupon rate) of bonds selling at par.

So if we know the par yield curve, we can figure out what is the correct coupon yield on a bond for it to sell at par.

In our example, we assume that we can extrapolate the par yield on a 10 year bond with credit quality identical to LIBOR banks. We can do this by constructing a par yield curve based on libor similar to fig. 18.2 in Tuckman.

The par yield on a 10 year bond was 5.668% on Nov 26, 2001.

Valuation of the floating rate notes

Notice that the floating rate payments are determined based on *previous* period's libor rate.

Suppose the libor rate is $L(T - .25)$ one quarter prior to expiration of the swap at time T . Then at time T , we get paid

$$100(1 + L(T - .25)\frac{90}{360})$$

at date T .

The present value of this payment is

$$\frac{100(1 + L(T - .25)\frac{90}{360})}{(1 + L(T - .25)\frac{90}{360})} = 100$$

at time $T - 0.25$ (one quarter before the final payment).

Since the discount factor is exactly the same as the interest payment, the floating rate note's value will always be par.

The combined cash flows from the fixed and long floating rate bonds are thus:

Table: Payments of fixed, floating rate notes and the difference (swap)

date	0	1/4	1/2	3/4	1	...	T
fixed	-100	0	$\frac{c}{2}$	0	$\frac{c}{2}$...	$100 + \frac{c}{2}$
floating	-100	$\frac{L(0)}{4}$	$\frac{L(\frac{1}{4})}{4}$	$\frac{L(\frac{1}{2})}{4}$	$\frac{L(\frac{3}{4})}{4}$...	$100 + \frac{L(T-\frac{1}{4})}{4}$
fixed-float	0	$-\frac{L(0)}{4}$	$\frac{c}{2} - \frac{L(\frac{1}{4})}{4}$	$\frac{L(\frac{1}{2})}{4}$	$\frac{c}{2} - \frac{L(\frac{3}{4})}{4}$...	$\frac{c}{2} - \frac{L(T-\frac{1}{4})}{4}$

The SWAP rate is basically just the YTM/coupon rate of a bond with credit quality similar to Libor banks.

Therefore, we can define the SWAP spread to be the difference between swap and treasury rates of similar maturity.

Finding the swap curve from zero curve

For the fixed leg to be selling at par, the par yield on a T maturity, c_T must satisfy

$$100 = \sum_{i=1}^{2T} d(i/2) \frac{c}{2} + 100d(2T)$$

where $d()$ is the discount function.

It is now easy to show that the par (swap) yield y_T must satisfy

$$c_T = 2 \times 100 \left(\frac{1 - d(2T)}{\sum_{i=1}^{2T} d(i)} \right)$$

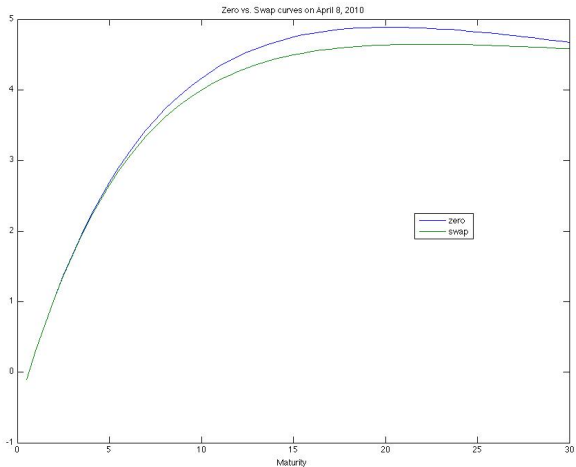


Figure: Zero vs. swap curves on April 8, 2010. Computed from NS fit.

Some swaps provide a spread over the libor. For example, the floating recipient gets LIBOR + X basis point. If so, we need to add X basis points to the fixed rate leg as well. The swap then pays

$$L(t - 0.25)/4 + X/4 - c/2 - X/2$$

every 6 m and

$$L(t - 0.25)/4 + X/4$$

every quarter when the fixed leg is zero.

If both paid interest at the same time, the extra interest would cancel exactly. With quarterly and semi-annual payments, they approximately cancel...

Assume a flat swap curve at 6.1%.

Consider the following trade:

- buy 100M worth of FNMA 6.25 of May 15, 2029 at par
- Repo out the FNMA bond
- enter a swap to pay 6.25 (6.1+15 BP) to receive LIBOR+15 BP

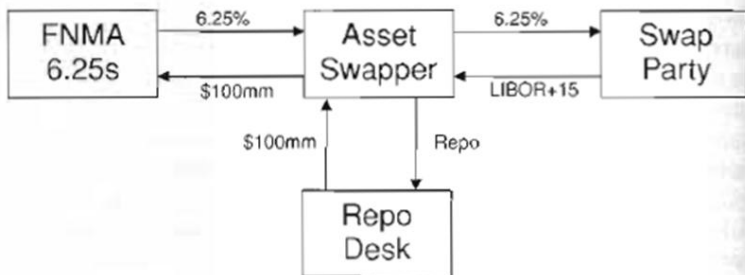


FIGURE 18.4 Asset Swap of the FNMA 6.25s of May 15, 2029

Net effect of this trade: Trader receives LIBOR+15 BP and pays repo. This is a good trade if the repo is below LIBOR + 15 BP.

Is it an arbitrage?

While swaps are basically designed to be buy-and-hold strategies, what happens if you opt out of the arrangement before the end of the contract?

You have to buy out the counterparty. Suppose we can do this for an exchange of cash. What is a fair exchange?

Suppose we receive fixed and pay floating. In this case, we are long a fixed rate coupon bond which market value is given as the present value of the fixed coupon and principal. We are short a floating rate bond that is always worth par.

Therefore: We must compensate the swap counterparty an amount equal to the difference between the current market value of the fixed leg (i.e., the coupon bond value), and par.

If interest rates increase, the fixed rate receiver in the swap contract has a capital (paper) loss. The fixed rate payer has a capital gain. And vice versa.

In other words, the fixed rate recipient has the same interest rate risk (i.e, duration) as that of a long bond holder. Swaps is a cheap way to bet on interest rate moves.

Define the asset swap spread of the FNMA asset swap trade to be the difference between the LIBOR + 15 BP and the FNMA repo rate. Since both are short rates, there is no law of physics that will bound the repo rate to be below the LIBOR+15.

Clearly, if the asset swap spread becomes negative, the trade loses money. Figure 18.6 in T plots the spread and shows that the repo was substantially below the LIBOR+15 throughout 2000, but then went above LIBOR+15 in 2001. The asset swap trade thus lost money.

There are two possible explanations:

- The market considered the credit quality of FNMA to be worse than that of the average bank surveyed in the LIBOR pool.
- Liquidity in the FNMA bond was low so that trading desks were charging a steep repo rate (liquidity premium)

Price Risks III: Collateral calls

Lets consider a parallel shift in all the interest rates in the previous example. Consider a X basis points increase in both the FNMA ytm, the LIBOR, and the repo rates.

The rate increase has the following effects on the trade:

- Both interest income (from the LIBOR + 15 bp) and expense (from the repo loan) increase by X bp and thus cancel
- The value of the FNMA bond decreases.

The problem here is that if the value of the collateral (FNMA bond) in the repo trade decreases, the trader might face a margin or collateral call from his repo counter-party.

According to Tuckman, the combination of the reversal of the spread, decrease in the value of the FNMA bond, forced liquidation of the asset swap trades which led to further deterioration in the value of the FNMA bonds.

Credit risk is not as severe of an issue as with risky debt per se because no principal ever changes hands.

Suppose, for the sake of argument, that the swap rates have not change since initiation of the the swap contract. In this case we stand to lose a maximum of one period interest payment.

In the case of an interest rate decrease (increase), the fixed recipient also loses (gains) an amount equal to the difference between market value of the fixed, and par.

Hedging with Swaps

Suppose an investor wishes to hedge a bond portfolio worth 10.2 M and with a modified duration of 8.35 with an interest rate swap.

Suppose further that the yield curve is flat at 5% and that he uses a plain vanilla swap with 10 year maturity and a notional amount of 1 M. The swap's modified duration is 7.72 - the same as a 5% coupon selling at par (..remember that the interest rate sensitivity of a swap equals that of its fixed leg)

The hedge ratio is

$$\frac{\$dur_p}{\$Dur_S} = -\frac{10.2}{1} \times \frac{8.35}{7.72} \approx -11$$

so we should sell 11 swaps to immunize the portfolio.

Basis swaps

Floating for floating rate swaps.

Ex 1: 6M LIBOR for 3M CD rates.

Ex 2. 3M LIBOR for 6M LIBOR

Accrediting, amortizing and roller coasters

Accrediting swap: Notional amount increases over time

Amortizing swap: Notional amount decreases over time

Roller coaster: Notional amount may rise or fall over time

Constant maturity swaps

Swaps LIBOR for a certain maturity swap rate.
(... chew on this...)

Constant maturity treasury swaps

Swaps a constant maturity treasury yield against constant maturity swap rate