Housing Quality, Housing Development and Public Policy

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Abstract: We construct a general equilibrium model with endogenous housing quality to augment the housing stock where the provision and finance of local public amenities play a crucial role. We consider an array of housing-related tax policies, including a developer gross revenue tax, a property tax, a land tax and a development license fee. In a competitive spatial equilibrium, all households optimize and reach the same utility in locational equilibrium, all monopolistically competitive developers optimize and receive zero profit, and both housing and land markets clear. We characterize the schedules of housing quality, housing prices, land rent, as well as the population and housing density. Moreover, we evaluate the effects of various tax policies on the housing market, the urban structure, and the welfare of the local economy.

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1 Introduction

The study of housing markets has long been a central issue in urban economics and regional science. In addition to the quantity of the housing stock, housing quality has played an important role in influencing consumer demand, housing and land prices, and the associated urban structure. Such quality measures may include the quality of the lot and the existing dwelling (including the vintage and the exterior/interior features of the dwelling), the spending on maintenance, repairs and improvements, as well as the quality of public/private amenities and the local environment. The American Housing Survey (1997, 1999, 2001, 2003 and 2005) indicates that there exist upward trends in housing quality, housing values (measured by purchase prices) and monthly housing costs (including mortgages and maintenance costs). Moreover, owner occupied newer units (4 years or less) in Metropolitan Statistical Area (MSA) suburbs feature higher housing quality and values than other units. For example, based on the 2005 survey, we find newer units in MSA suburbs are most likely to have porch (or deck, patio, or balcony), fireplace, and garage, and to have separate dining room and multiple living or recreation rooms, while older units in MSA central cities are least likely to have such features. We also find housing quality and housing values are positively related.

	All Units			Newer Units		
	All Areas	MSA		All Areas	MSA	
		Central	Suburb		Central	Suburb
Median Housing Value	$165,344	$161,096	$205,218	$238,365	$241,366	$256,556
Median Housing Costs	$853	$882	$1,029	$1,299	$1,390	$1,394
Proportions (%) of Units Featuring						
porch	83.7	76.0	87.8	87.7	84.8	91.2
fireplace	32.8	25.6	40.5	47.8	42.0	53.6
garage	59.8	51.6	66.7	77.6	76.1	81.1
separate dining rooms	46.3	44.0	50.9	53.9	50.8	57.2
multiple living rooms	27.6	19.8	33.4	41.0	34.3	45.9

The main purpose of this paper is to extend the housing quality literature by considering both the demand and the supply factors to determine housing quality and by incorporating the provision and finance of local public amenities as a key element affecting housing quality and housing prices. This framework is then used to evaluate the effects of an array of housing-related tax instruments, including a developer gross revenue tax, a property tax, a land tax and a development license fee (development tax), on the housing market, the urban structure, as well as the welfare of the local economy.

Specifically, we develop a general equilibrium model of housing quality where the quality, quantity and quality-augmented price are all endogenously determined. Concerning housing quality, we focus exclusively on the exterior and interior features of the dwelling as well as the congestion aspect of the local environment.[1] We consider two special features: (i) the provision and finance of local public amenities are important for households' demand for housing quality and (ii) local monopoly power and local housing congestion are key elements driving developers' housing supply. While all households reach the same utility in locational equilibrium, all developers receive zero profit under free entry in a monopolistic competition framework. In competitive spatial equilibrium, the land rent, the population density and the size of the city are also pinned down. The model enables us to address two important issues: (i) how housing quality/quantity/price and urban structure respond to changes in preference, technology and policy parameters, and (ii) which tax policy mix lead to higher welfare for the local economy.

The main findings are summarized as follows. First, while the population density, the land rent, and the number of floors (housing quantity) are decreasing in the distance away from the city center, housing quality and quality-augmented housing prices (housing values) are increasing in it. The latter result is consistent with the empirical observation in the U.S. Second, in response to any changes in preferences and in production, commuting and development technologies, housing quality and housing prices/values are positively related, corroborating with the empirical fact. Specifically, an increase in households' preference bias toward housing quality or developers' productivity of housing quality, or a decrease in the fixed cost of housing development generally raises housing quality and housing prices, expands the city fringe, and leads to a flatter schedule of the population density and the land rent gradient. A decrease in the unit commuting cost reduces the number of floors and raises housing quality and prices in the inner city, but generates a reverse outcome in the outer city. Finally, based on our quantitative welfare analysis, a globally optimal tax scheme in the housing market (without income or distortionary factor taxes) is to eliminate the property tax and to impose a lower gross revenue tax rate than either the development or the

[1] Since our model is static, we do not consider the age of the structure or the improvement in durable housing over time. Moreover, while we do consider public amenities, we do not include their services into the measure of housing quality. We will relegate the discussion of such a possibility to the concluding section.

land tax.

Literature Review

There is a conventional literature studying urban land use and durable housing pioneered by Arnott (1980), Brueckner (1981) and Fujita (1982), where housing quality is not fully characterized. In his pivotal work, Sweeney (1974) formalizes the notion of housing quality. Since then, there has been numerous empirical studies estimating housing demand and housing prices in which housing quality plays a crucial role.

Theoretically, we are only aware a few papers examining the equilibrium determination of housing quality. In particular, Arnott, Davidson and Pines (1983) develop a partial-equilibrium model of housing quality from the supply side based on profit maximization of the representative landlord. More recently, Arnott, Braid, Davidson and Pines (1999) construct a general-equilibrium housing model with households choosing quality and quantity of housing and developers determining the structure density (and time path) of housing. Lin, Mai and Wang (2004) establish an endogenous growth model in which housing quality measured by housing capital is regarded as an intermediate input to enhance household production and quality-augmented leisure time.[2]

In contrast to the above studies, our paper considers both the demand and the supply factors to determine housing quality and allows quality, quantity and quality-augmented price to be jointly determined in a general equilibrium framework. Moreover, we model explicitly the provision and finance of local public amenities. It is noted that, while the housing quality and urban structure literature summarized above does not consider the role of local public good services, previous studies of housing demand and public goods (e.g., Turnovsky and Okuyama, 1994, Brueckner, 2001, and Sieg, Smith, Banzhaf and Walsh, 2004) do not take into account the explicit spatial structure of the local economy. Furthermore, in contrast with both lines of research work, our paper allows developers to have local monopoly power and their housing quality supply to be subject to local congestion.

2 The Model

Consider a "long-narrow" (linear) city spread over a featureless, uniformly distributed land $[-x_f, x_f]$, where the urban fringe x_f is endogenously determined. There is an exogenously determined city

[2] There is a long list of studies along these lines, which are not discussed here for the sake of brevity. Those interested may be referred to papers cited in Arnott et al (1999) and Lin et al (2004).

center at location 0 in which a "travel-for" local public good (LPG) is established. There are three theatres of economic activities, played by households, housing developers and the local government.

2.1 Households

There is a continuum of households of measure N with identical income, y, and identical preferences. All households commute to the central business district (CBD) to work (with an inelastic labor supply), to shop, and to enjoy local public amenities by incurring a commuting cost at a constant rate t per unit of distance. More precisely, households must travel to the CBD to consume the LPG service and the LPG is both non-rival and non-exlusive.

By residing in a house whose quantity is fixed at one unit, a representative household at location z values composite good consumption, c, housing quality, q, and the level of the LPG (G) provided by the local government.[3] Her utility function takes the following form:

$$U(c, q, z) = c + \beta q^{\theta} + \gamma \ln G \tag{1}$$

where β measures the household's preference toward housing quality, γ indicates her preference toward the LPG and $\theta \in (0, 1)$. This setup extends the quasi-linear utility function form employed by Bergstrom and Cornes (1983) and Berliant, Peng and Wang (2006). When $\beta = 0$, the utility function reduces to that in Berliant, Peng and Wang (2006).

Given the unit price associated with quality q at location z, $P(q, z)$, and the local property tax rate, τ_H, the representative household allocates her net income (income net of the commuting cost incurred) to consumption of the composite good and housing (inclusive of the property tax payment). Her budget constraint can therefore be written as:

$$c + (1 + \tau_H)P(q, z)q = y - tz \tag{2}$$

The representative household's optimization problem is thus to choose composite good consumption, housing quality and residential location, (c, q, z), to maximize U given in (1) subject to the budget constraint (2).

Straightforward manipulation of the first-order conditions with respect to c and q yields:

$$P(q, z) = \frac{\beta\theta}{1 + \tau_H} q^{\theta - 1} \tag{3}$$

[3] On the demand side, our setup of housing quantity and housing quality is similar to that in Tanzer (1985), in which housing quality enters households' preferences with an inelastic demand for a housing quantity of one unit. Yet, Tanzer does not take an explicit account for the spatial structure.

The envelope condition for z (i.e., the Muth condition) gives:

$$[\beta\theta q^{\theta-1} - (1+\tau_H)P(q,z)]\frac{dq}{dz} - (1+\tau_H)P_z q - (1+\tau_H)P_q q\frac{dq}{dz} = t \tag{4}$$

Using (3), we can rewrite (4) as:

$$(P_z + P_q\frac{dq}{dz}) = -\frac{t}{(1+\tau_H)q} \tag{5}$$

Totally differentiating (3) and combining it with (5), we have:

$$q^{\theta-1}\frac{dq}{dz} = \frac{t}{\beta\theta(1-\theta)}$$

Straightforward integration of this first-order differential equation yields the demand schedule for housing quality:

$$q(z) = q^d(z) \equiv \left[q_0 + \frac{t}{\beta(1-\theta)}z\right]^{\frac{1}{\theta}} \tag{6}$$

where the integration constant, q_0, measures the housing quality at location 0 which will be endogenously determined in equilibrium. Thus, housing quality demand is increasing in the distance away from the CBD.

Substituting (6) into (3), we obtain the unit bid price schedule associated with housing quality, which is decreasing in z:

$$p(z) \equiv P(q(z),z) = \frac{\beta\theta}{1+\tau_H}\left[q_0 + \frac{t}{\beta(1-\theta)}z\right]^{-\frac{1-\theta}{\theta}} \tag{7}$$

Furthermore, the before-tax bid price augmented by quality at each location z (associated with a house of floor size one) can be derived as:

$$p(z)q(z) = \frac{\beta\theta}{1+\tau_H}\left[q_0 + \frac{t}{\beta(1-\theta)}z\right] \tag{8}$$

Thus, for given q_0, the before-tax bid price is increasing in z. This is because the housing quality effect always dominates the unit bid price effect at each location.

2.2 Developers

There is a continuum of developers of measure M, each developing a lot at a particular location. The land available at each location is one unit, which is owned entirely by an absentee landlord and rented to a developer (or housing producer). Under free entry and no vacant land, the endogenous measure of developers is given by, $M = 2x_f$.

Each developer employs capital per house k, together with one unit of land, to produce houses, whose quantity measured by the number of floors (the floor-area-ratio) is denoted by h. The developer at a given location has local monopoly power and compete with each other in a monopolistically competitive fashion.

We specify the housing quality production function as follows:

$$q(x) = Ak(x)^{\alpha} h(x)^{-\varepsilon} \tag{9}$$

where $A > 0$, $0 < \alpha < 1$ and $\varepsilon > 0$. The production scaling parameter, A, measures the developer's productivity. The parameter ε captures the degree of housing congestion: the more houses per unit of land (or, precisely, more floors) developed in a given location, the less the quality of a given house is.

Denote $R(x)$ as the market land rent per unit of land at location x, r as the capital rental rate, and ϕ as the fixed lot-development fee (which is a sunk cost). Further denote the gross revenue tax rate as τ_S, the land tax rate as τ_L, and the development tax rate as τ_D (which is the percentage licence fee taxed on the basis of the fixed development cost). The profit generated by the housing developer at location x is then given by:

$$\pi(x) = (1 - \tau_S)\, P(q(x), x) q(x) h(x) - rk(x)h(x) - (1 + \tau_L)\, R(x) - (1 + \tau_D)\phi \tag{10}$$

which equals the after-tax revenue, net of capital costs, land rents, and tax-included development fees. Thus, a housing developer's optimization is to determine capital demand, housing quantity supply and housing development location, (k, h, x), to achieve maximum profit, or, equivalently, to pin down profit-maximizing capital demand, housing quality supply and the housing development location. Notably, capital is the only elastic factor input, which is perfectly mobile. Thus, there is no need for deriving the envelope condition for x because it is assured by the free entry condition. Also, to avoid complexity from the joint production problem (of housing quantity and housing quality), our setup allows us to first pin down capital demand and then adjust housing quantity to meet household's housing quality demand.

Substituting the inverse demand function (3) and the production function (9) into developers' profit function (10) yields:

$$\pi(x) = \left(\frac{1 - \tau_S}{1 + \tau_H} \beta\theta A^{\theta} k^{\alpha\theta} h^{-\varepsilon\theta} - rk \right) h - (1 + \tau_L)\, R(x) - (1 + \tau_D)\phi \tag{11}$$

From the first-order condition with respect to k, one gets:

$$k = \left[\left(\frac{1-\tau_S}{1+\tau_H} \right) \frac{\alpha\beta\theta^2 A^\theta}{r} \right]^{\frac{1}{1-\alpha\theta}} h^{-\frac{\varepsilon\theta}{1-\alpha\theta}} \tag{12}$$

Substituting (12) into (9), we have

$$q^s(x) = A^{\frac{1}{1-\alpha\theta}} \left[\left(\frac{1-\tau_S}{1+\tau_H} \right) \frac{\alpha\beta\theta^2}{r} \right]^{\frac{\alpha}{1-\alpha\theta}} h^{-\frac{\varepsilon}{1-\alpha\theta}} \tag{13}$$

from which we can derive the quality-augmented housing stock $h^s(x) \equiv q(x)h(x)$. A quick observation suggests that, other things being equal, housing quantity measured by the floor-area-ratio and housing quality supply are negatively related.

2.3 Local Government

Consider a local government to provide public amenities at the CBD for all households residing in the city. Denote the level of the LPG provision as G. We suppose that this LPG is entirely financed by revenues collected from the four housing-related taxes: the gross revenue tax, the property tax, the land tax, and the development tax. Then balancing the government budget requires:

$$G = 2(\tau_H + \tau_S) \int_0^{x_f} p(z)h^s(z)dz + 2\tau_L \int_0^{x_f} R(z)dz + \tau_D \phi M \tag{14}$$

(recall that $M = 2x_f$). It is easily seen that while the land tax and the development tax have different tax bases than the gross revenue tax and the property tax, the latter two have an identical tax base, $2 \int_0^{x_f} p(z)h^s(z)dz$. Thus, if the gross revenue tax and the property tax may affect the equilibrium outcomes differently, it must be due exclusively to differences in their marginal distortions to demands and supplies (and hence the associated prices).

3 Equilibrium

In a *competitive spatial equilibrium*, all households and developers optimize, all households reach the same indirect utility, all developers reach zero profit, the government budget is balanced, and the housing markets (quantity and quality) and the land market all clear. The solution method is outlined as follows.

- Step 1: We use the housing quality market clearing condition at each location and the zero profit condition to express housing quantity and market land rent, respectively, as a function of q_0.

- Step 2: Substituting housing quantity and land rent schedules into the land market clearing condition, the housing quantity clearing condition at each location, and the population identity, we can then determine the equilibrium housing quantity schedule as well as the urban fringe and the value of q_0.

- Step 3: Substituting q_0 into the housing quality demand schedule, the unit bid price schedule, and the land rent schedule yield the corresponding equilibrium schedules as well as the equilibrium schedules of the (quality-augmented) housing stock and the (quality-augmented) price.

- Step 4: Substituting the above equilibrium outcomes into the government budget constraint as well as the household budget constraint and utility function, one obtains the equilibrium provision of the LPG (for given tax rates), the equilibrium consumption schedule and the constant value of indirect utility across all locations.

3.1 Step 1

The clearance of the housing market from the quality perspective at each location x in equilibrium is:

$$[q_0 + \frac{t}{\beta(1-\theta)}x]^{\frac{1}{\theta}} = A^{\frac{1}{1-\alpha\theta}} \left[\left(\frac{1-\tau_S}{1+\tau_H} \right) \frac{\alpha\beta\theta^2}{r} \right]^{\frac{\alpha}{1-\alpha\theta}} h^{-\frac{\varepsilon}{1-\alpha\theta}} \tag{15}$$

Note that housing quality demand (the lefthand side) is independent of housing quantity whereas housing quality supply (the righthand side) depends negatively on housing quantity. Thus, for given q_0, one can plot quality demand and supply at each location z in Figure 1 (the right panel) to pin down housing quantity as a function of q_0 (see point E in Figure 1), which is given by,

$$h(x) = A^{\frac{1}{\varepsilon}} \left[\left(\frac{1-\tau_S}{1+\tau_H} \right) \frac{\alpha\beta\theta^2}{r} \right]^{\frac{\alpha}{\varepsilon}} \left[q_0 + \frac{t}{\beta(1-\theta)}x \right]^{-\frac{1-\alpha\theta}{\varepsilon\theta}} \tag{16}$$

Then substituting (16) into (12) we have:

$$k(x) = \left[\left(\frac{1-\tau_S}{1+\tau_H} \right) \frac{\alpha\beta\theta^2}{r} \right] \left[q_0 + \frac{t}{\beta(1-\theta)}x \right] \tag{17}$$

From (16) and (17), one can see that housing quantity measured by the floor-area ratio is decreasing in the distance away from the CBD whereas the demand for capital per unit of house is increasing in it.

Zero profit under free entry implies:

$$R(x) = \frac{1-\alpha\theta}{1+\tau_L} A^{\frac{1}{\varepsilon}} \left(\frac{\alpha\theta}{r}\right)^{\frac{\alpha}{\varepsilon}} \left[\left(\frac{1-\tau_S}{1+\tau_H}\right)\beta\theta\right]^{\frac{\alpha+\varepsilon}{\varepsilon}} \left[q_0 + \frac{t}{\beta(1-\theta)}x\right]^{-\frac{1-(\alpha+\varepsilon)\theta}{\varepsilon\theta}} - \left(\frac{1+\tau_D}{1+\tau_L}\right)\phi \quad (18)$$

We impose a "normality" condition to guarantee that a higher development fee will reduce the (quality-augmented) housing stock $h^s(x) \equiv q(x)h(x)$ (to be formally proved later):[4]

Condition N: $\alpha\theta + \varepsilon < 1$.

This assumption is sufficient to guarantee $(\alpha + \varepsilon)\theta < 1$, which ensures that the market land rent is decreasing in x (a standard feature of monocentric city models).

3.2 Step 2

The main task now is to pin down a key endogenous variable, q_0. Land market clearance implies that the linear city continues to be developed until the market land rent at the endogenously determined fringe x_f equals the agricultural land rent, R_A:

$$R(x_f) = R_A \quad (19)$$

Using (18) and (19), we can solve q_0 as a decreasing function of the city fringe x_f:

$$q_0 = B - \frac{t}{\beta(1-\theta)}x_f \quad (20)$$

where $B(\beta, A, \phi, \tau_H, \tau_S, \tau_L, \tau_D) \equiv \left\{\frac{(1-\alpha\theta)A^{\frac{1}{\varepsilon}}\left(\frac{\alpha\theta}{r}\right)^{\frac{\alpha}{\varepsilon}}\left[\left(\frac{1-\tau_S}{1+\tau_H}\right)\beta\theta\right]^{\frac{\alpha+\varepsilon}{\varepsilon}}}{(1+\tau_D)\phi+(1+\tau_L)R_A}\right\}^{\frac{\varepsilon\theta}{1-(\alpha+\varepsilon)\theta}}$, with $\frac{\partial B}{\partial \beta} > 0$, $\frac{\partial B}{\partial A} > 0$, $\frac{\partial B}{\partial \phi} < 0$, $\frac{\partial B}{\partial \tau_H} < 0$, $\frac{\partial B}{\partial \tau_S} < 0$, $\frac{\partial B}{\partial \tau_L} < 0$, and $\frac{\partial B}{\partial \tau_D} < 0$. That is, other things being equal, a larger city will feature houses in the inner city with lower quality than a smaller city.

Furthermore, since each household owns exactly one unit of house (i.e., one floor of the dwelling at a particular lot), housing market clearance from the quantity perspective implies that the population density at each location is equal to the floor-area-ratio: $n(x) = h(x)$. Substituting (20) into (16) implies:

$$n(x) = h(x) = A^{\frac{1}{\varepsilon}} \left[\left(\frac{1-\tau_S}{1+\tau_H}\right)\frac{\alpha\beta\theta^2}{r}\right]^{\frac{\alpha}{\varepsilon}} \left[B - \frac{t}{\beta(1-\theta)}(x_f - x)\right]^{-\frac{1-\alpha\theta}{\varepsilon\theta}} \quad (21)$$

[4] This normality condition is essentially Samuelson's correspondence priciple: it ensures stability and reasonable comparative statics.

which is decreasing in the distance from the CBD. Since there are totally N households residing within the geographically symmetric linear city $[-x_f, x_f]$, the following population identity must be met:

$$2\int_0^{x_f} h(x)dx = N \tag{22}$$

which in turn pins down the urban fringe. Specifically, one can plug (21) into (22) and integrate it to obtain:

$$x_f = \frac{\beta(1-\theta)B}{t}\left[1-\left(1+D\frac{N}{2}\right)^{\frac{-\varepsilon\theta}{1-(\alpha+\varepsilon)\theta}}\right] \tag{23}$$

where $D(\phi, t, \tau_H, \tau_S, \tau_L, \tau_D) \equiv \frac{\left(\frac{1-\tau_S}{1+\tau_H}\right)t[1-(\alpha+\varepsilon)\theta](1-\alpha\theta)}{\varepsilon(1-\theta)[(1+\tau_D)\phi+(1+\tau_L)R_A]}$, with $\frac{\partial D}{\partial \phi} < 0$, $\frac{\partial D}{\partial t} > 0$, $\frac{\partial D}{\partial \tau_H} < 0$, $\frac{\partial D}{\partial \tau_S} < 0$, $\frac{\partial D}{\partial \tau_L} < 0$, and $\frac{\partial D}{\partial \tau_D} < 0$. Thus, a shift in households' preferences toward housing quality (higher β), an increase in developers' productivity (A), or a reduction in the development fee (ϕ) will all enable developers at the outskirts to maintain profitability more easily, thereby enlarging the urban fringe. Conversely, an increase in any tax rate leads to a smaller city. Since there is only one developer at each location, the endogenous number of developers is now pinned down by: $M = \frac{2\beta(1-\theta)B}{t}\left[1-\left(1+D\frac{N}{2}\right)^{\frac{-\varepsilon\theta}{1-(\alpha+\varepsilon)\theta}}\right]$.

We can then substitute (23) into (21) and use (20) (and change uniformly the location index in equilibrium to z) to derive:

$$n(z) = h(z) = A^{\frac{1}{\varepsilon}}\left[\left(\frac{1-\tau_S}{1+\tau_H}\right)\frac{\alpha\beta\theta^2}{r}\right]^{\frac{\alpha}{\varepsilon}}\left[q_0 + \frac{t}{\beta(1-\theta)}z\right]^{-\frac{1-\alpha\theta}{\varepsilon\theta}} \tag{24}$$

where

$$q_0(\beta, A, \phi, t, \tau_H, \tau_S, \tau_L, \tau_D) = \left[\frac{A\left(\frac{\alpha\theta}{r}\frac{1-\tau_S}{1+\tau_H}\right)^{\alpha}(\beta\theta)^{\alpha+\varepsilon}}{\left\{\frac{(1+\tau_D)\phi+(1+\tau_L)R_A}{(1-\alpha\theta)\frac{1-\tau_S}{1+\tau_H}} + \frac{tN[1-(\alpha+\varepsilon)\theta]}{2\varepsilon(1-\theta)}\right\}^{\varepsilon}}\right]^{\frac{\theta}{1-(\alpha+\varepsilon)\theta}} \tag{25}$$

with $\frac{\partial q_0}{\partial \beta} > 0$, $\frac{\partial q_0}{\partial A} > 0$, $\frac{\partial q_0}{\partial \phi} < 0$, $\frac{\partial q_0}{\partial t} < 0$, $\frac{\partial q_0}{\partial \tau_H} < 0$, $\frac{\partial q_0}{\partial \tau_S} < 0$, $\frac{\partial q_0}{\partial \tau_L} < 0$, and $\frac{\partial q_0}{\partial \tau_D} < 0$.

3.3 Step 3

We are now ready to substitute (25) into (6), (7), (23), and (18) to obtain:

$$q(z) = \left[q_0(\bullet) + \frac{t}{\beta(1-\theta)}z\right]^{\frac{1}{\theta}} \tag{26}$$

$$p(z) = \frac{\beta\theta}{1+\tau_H}\left[q_0(\bullet) + \frac{t}{\beta(1-\theta)}z\right]^{-\frac{1-\theta}{\theta}} \tag{27}$$

$$x_f = \frac{\beta(1-\theta)}{t}\left[B - q_0(\bullet)\right] \tag{28}$$

$$R(z) = \frac{(1+\tau_D)\,\phi + (1+\tau_L)\,R_A}{1+\tau_L}\left[\left(1+D\frac{N}{2}\right)^{\frac{-\varepsilon\theta}{1-(\alpha+\varepsilon)\theta}} + \frac{t}{\beta B(1-\theta)}z\right]^{-\frac{1-(\alpha+\varepsilon)\theta}{\varepsilon\theta}} - \frac{1+\tau_D}{1+\tau_L}\phi \tag{29}$$

As it can be seen clearly from (26), equilibrium housing quality depends on two components: (i) the first component, q_0, indicates the *overall quality* of houses in the city that is independent of z and (ii) the second component, $\frac{t}{\beta(1-\theta)}z$, measures the *quality gradient* that is increasing in z. However, housing quality is not linear in the two components ($\theta < 1$). Thus, although an overall quality enhancement (an increase in q_0) raises the equilibrium quality of houses in all locations, it raises the quality of houses at the outskirts more than those in the inner city. As a consequence, an increase in any tax rate, which affects housing quality only via q_0 (see (25)), will create a larger distortion to the quality of houses at outskirts. Additionally, from (27) and (28), an increase in the overall quality will reduce both the unit bid price (per unit of quality) and the urban fringe.

Furthermore, we can combine (24) and (26) to derive the equilibrium housing stock (quality augmented) at location z:

$$h^s(z) \equiv q(z)h(z) = A^{\frac{1}{\varepsilon}}\left[\left(\frac{1-\tau_S}{1+\tau_H}\right)\frac{\alpha\beta\theta^2}{r}\right]^{\frac{\alpha}{\varepsilon}}\left[q_0(\bullet) + \frac{t}{\beta(1-\theta)}z\right]^{-\frac{1-(\alpha\theta+\varepsilon)}{\varepsilon\theta}} \tag{30}$$

Therefore, under Condition N, the equilibrium housing stock is decreasing in the distance away from the CBD. Next, denote by $v(z) \equiv p(z)q(z)$ the equilibrium housing price (quality augmented), which also measures the equilibrium housing value. From (3), the equilibrium housing price must equal the corresponding bid price augmented by quality at location z:

$$v(z) = P(q(z), z)q(z) = \frac{\beta\theta}{1+\tau_H}q(z)^{\theta} \tag{31}$$

which is increasing and strictly concave in q. This locus, as well as the downward sloping unit bid price locus (3), can be plotted in the left panel of Figure 1, which can then be combined with the right panel to pin down equilibrium housing quantity, housing quality, unit bid price, and housing values. To obtain a close-form solution, we utilize (26) and (27) to get:

$$v(z) = \frac{\beta\theta}{1+\tau_H}\left[q_0(\bullet) + \frac{t}{\beta(1-\theta)}z\right] \tag{32}$$

which is unambiguously increasing in z. Interestingly, the equilibrium housing price/value turns out to be linear in q_0. Thus, an increase in q_0 raises housing prices uniformly across all locations and, other than the property tax that has a direct negative effect on housing prices, the other three tax instruments will all reduce housing prices uniformly across all locations (by affecting q_0).

3.4 Step 4

Finally, from (2), (3), we have: $c(z) = y - tz - (1+\tau_H)\beta\theta q^\theta$, which together with (6) yields,

$$c(z) = y - tz - (1+\tau_H)\beta\theta \left[q_0(\bullet) + \frac{t}{\beta(1-\theta)} z \right]^\theta \tag{33}$$

Substituting (33) into (1), we obtain the indirect utility:

$$u(z) \equiv U(c(z), q(z), z) = y - tz + \beta\left[1-(1+\tau_H)\theta\right] \left[q_0(\bullet) + \frac{t}{\beta(1-\theta)} z \right]^\theta + \gamma \ln G(\bullet)$$

where the equilibrium provision of LPG, $G(\bullet)$, is pinned down by plugging (27), (29) and (30) into (14) (which is not fully spelled out for the sake of brevity). Since $u(z) = u(0)$ for all z within the entire city, the equilibrium utility level can be computed as:

$$u(0) = y + \beta\left[1-(1+\tau_H)\theta\right]\left[q_0(\bullet)\right]^\theta + \gamma \ln G(\bullet) \tag{34}$$

4 Equilibrium Characterization

We are now ready to perform comparative-static exercises. We consider shifts in preference parameter (β), production parameter (A), development fee parameter (ϕ) and commuting cost parameter (t), as well as the four tax policy parameters (τ_H, τ_S, τ_L, τ_D). We are interested in how such parameter changes affect an array of endogenous variables, including x_f (city fringe), $q(z)$ (housing quality), $h^s(z)$ (housing stock), $v(z)$ (housing price/value), $n(z)$ (population density), $R(z)$ (land rent), and $u(0)$ (household welfare), which are determined by (28), (26), (30), (32), (24), (29), and (34), respectively.

4.1 Analytical Results

The analytical results are summarized as follows:

effect of	x_f	q_0	$q(z)$	$v(z)$	$h^s(z)$	$u(0)$
β	+	+	?	+	?	+
A	+	+	+	+	+	+
ϕ	−	−	−	−	−	−
t	?	−	?	?	?	?
τ_H	−	−	−	−	?	−
τ_S	−	−	−	−	?	−
τ_L	−	−	−	−	+	−
τ_D	−	−	−	−	+	−

The results are best illustrated by utilizing Figure 1. First, we begin with the most straightforward case concerning a reduction in the unit development fee (ϕ), which only creates an overall quality enhancement effect via q_0. Specifically, it causes the quality demand schedule to shift up without affecting the quality supply schedule. As a result, the equilibrium shifts from point E to point E', which features higher housing quality, lower housing quantity (floors per location) and higher housing prices/values. Intuitively, a lower development fee enables firms in the outer city to maintain profitability more easily, thus leading to a larger city with lower density of houses (less floors) at each location (see (23) and (24), respectively). The reduction in housing congestion in turn improves the overall quality of houses in the city. Notice that these changes are simply along the housing quality supply curve. To restore housing quality equilibrium, housing quality demand must shift up, as indicated in the right panel of Figure 1. Since the positive quality effect dominates the negative quantity effect under Condition N, the equilibrium quality-augmented housing stock turns out to be higher. Moreover, as indicated in the left panel of Figure 1, while the unit bid prices decrease for every location in the city, quality-augmented housing prices (housing values) are always higher because the quality effect dominates.

Second, improved productivity of housing quality (A) creates similar effects via overall quality as a reduction in the development fee. Additionally, it shifts the housing quality supply schedule outward (see Figure 1), which tends to increase housing quantity. Due to the presence of the congestion effect, however, this positive effect is outweighed by the negative effect as a result of the shift in housing quality demand. Thus, the equilibrium shifts from point E to point E'' and housing quantity decreases. Qualitatively, the comparative static results are therefore identical to those with respect to a reduction in the development fee.

Third, an increase in the preference bias toward housing quality (β) encourages the development of a larger city size. While it creates similar effects via overall quality and the outward shift in the housing quality supply schedule as improved productivity, there is an additional negative quality gradient effect. Specifically, in response to this preference shift, the urban fringe expands and the resulting commuting cost for households residing in the outer city increases, so their willingness to pay for housing quality lessens. This induces a downward shift in the housing quality demand schedule, which may outweigh the overall quality effect and hence its net effects on housing quality and quantity become ambiguous. It is therefore possible that housing quality and quantity are positively related in response a preference shift. Moreover, in response to this preference shift, the unit bid price locus shifts outward and the housing value locus rotates outward (see Figure 1). Thus, while the effect on housing quality is generally ambiguous, it always raises equilibrium housing values as a result of this dominating direct effect via the shift in the housing value locus.

Fourth, similar to the preference shift mentioned above, a reduction in the unit commuting cost (t) also has two opposing effects on housing quality: an overall quality enhancement effect and a negative quality-gradient effect. However, from (23), it generates two opposing effects on the urban fringe as well. The first is a *direct commuting* effect: cheaper commuting enables households to afford living farther from the city center, thereby causing the city to expand. The second is an indirect overall quality effect: cheaper commuting leads to housing quality enhancements even at locations near the center, thus discouraging households to move outward seeking houses of greater quality. As a consequence, an improvement in the commuting technology has an ambiguous effect on the urban fringe. This ambiguity leaves essentially all comparative static results indeterminate. In locations toward the city center, it is however expected that the overall quality enhancement effect dominates and hence housing quality improves unambiguously.

Finally, we examine the effects of the four tax instruments. Note that, qualitatively, land and development tax rates (τ_L and τ_D) generates similar effects as the development fee: a reduction in either tax rate shifts the housing quality demand schedule upward and results in higher housing quality, lower housing quantity and higher housing prices/values. In contrast, property and gross revenue tax rates (τ_H and τ_S) create additional distortions on capital demand (see (17)). A reduction in property or gross revenue tax rate lowers capital distortion, thus causing the housing quality supply schedule to shift outward. Although this seems to resemble an improvement in productivity, the presence of congestion can no longer guarantee that the quality supply effect is outweighed by the quality demand effect. As a consequence, equilibrium housing quantity at each location need not be lower. Furthermore, a reduction in the property tax rate also creates similar effects on the unit bid price and housing value loci as an increase in the preference bias toward housing quality.

4.2 Numerical Findings

We now turn to numerical analysis, which not only help clarify the ambiguity in analytical comparative statics but also quantify such responses. We set $\theta = 0.6$, $\varepsilon = 0.5$, $N = 1000$, and $y = 1$ (per capital income normalized as one). We choose $\alpha = 3/4$ based on the share of structure capital in housing production and $r = 7.5\%$ based on the U.S. average capital rental rate. The preference parameter is set at $\beta = 0.2$ such that the share of flow housing expenditure to income ($\frac{TGR}{Ny}$, where $TGR = 2\int_0^{x_f} p(z)q(z)h(z)zdz$ measures the total gross revenue from housing construction as well as the total flow housing expenditure) falls in the range between 5 and 10% (7.4% in our benchmark case). The unit commuting cost is set at $t = 0.04$ such that the share of commuting cost to household income ($\frac{ACC}{Ny}$, where $ACC = 2t\int_0^{x_f} h(z)zdz$ measures the aggregate commuting cost) takes a reasonable value about 4 to 5%.

Further, we select $\phi = 0.3$ for the ratio of the total development fee ($TDF = 2\phi x_f$) to total gross revenue (TGR) to fall in the range between 10 and 15% and $A = 10$ for the ratio of aggregate land rent to income ($\frac{ALR}{Ny}$) to take a reasonable value about 2.5%. In order for the ratio of agriculture land rent to the average city land rent ($\frac{R_A}{ALR/(2x_f)}$) to fall in the range between 10 and 15%, we set agricultural land rent as $R_A = 0.1$, from which we get $x_f = 14.7$ and $\frac{R_A}{ALR/(2x_f)} = 11.7\%$. Finally, we choose housing tax instruments mimicking a typical U.S. city, $(\tau_H, \tau_S, \tau_L, \tau_D) = (0.15, 0.075, 0.125, 0.08)$, which, together with $\gamma = 0.5$, generate a reasonable LPG to aggregate income ratio around 2%.[5]

[5] More specifically, we must bear in mind that, in a static framework, housing values should be measured in flows. Consider a house with a market value of \$100,000. Given a 7.5% rental rate, its flow value is \$7,500. Under a typical 1.5% property tax rate imposed on the assessed value where the assessed value is assumed about 75% of the market value, the property tax payment becomes $1.5\% \cdot 75\% \cdot \$100,000 = \$1,125$. Dividing this payment by the flow housing value \$7,500, one obtains an imputed property tax rate of 15%. Next, conerning the gross revenue tax, a 30% profit tax rate together with a 25% profit rate implies an imputed gross revenue tax at 7.5%. Moreover, because the average land rent to gross revenue ratio is 1/3, a land tax rate of 12.5% is equivalent to a reasonable 4% lot surcharge attached to the housing value. Finally, since the average development fee to gross revenue ratio is about 12%, a development tax of 8% is equivalent to a plausible 1% licensing surcharge attached to the housing value.

We present the numerical results with respect to preference (β) and technology (A, ϕ and t) parameters in Table 1; those with respect to tax policy parameters are reported in Table 2. The main findings are summarized as follows. First, it is generally true that the direct effect on housing quality and the housing stock dominates the indirect effect via household's willingness to pay.[6] Second, a lower unit commuting cost increases the city size which suggests that the direct commuting effect dominates the indirect overall quality effect. Third, a reduction in either property or gross revenue tax rate lowers housing quantity at each location, implying that the quality demand effect dominates the quality supply effect.

Fourth, we can fully characterize how the population density, housing quality, housing price, housing stock and land rent schedules changes in responses to shifts in preference and technology parameters. The results are conveniently plotted in Figures 2 and 3. Briefly, an increase in β or A, or a decrease in ϕ expands the city fringe and encourages households to relocate from the inner to the outer city. As a result, both the population density and the land rent gradient are flatter, though the land rent at the CBD remains unchanged. In contrast, a decrease in the unit commuting cost not only flattens the population density and the land rent gradient but also reduces the land rent at the CBD.

Fifth, it is interesting that the effects of a reduction in the unit commuting cost (t) on the housing stock, housing quality and housing prices depend crucially on the location of the house. In the inner city, lower commuting cost reduces the number of floors and raises housing quality and prices; such results are reverse in the outer city. Accordingly, a rapid increase in commuting costs may result in a quick deterioration of the inner city from the aspect of housing quality and housing prices.

Finally, the city size, the population density, housing quality and housing prices are most responsive to changes in the preference parameter (β), followed by the production and commuting technology parameters (A and t). While the number of floors and the housing stock are most responsive to the production technology parameter (followed by the preference parameter), the land rent is only responsive to the commuting cost parameter. Development cost parameter creates relatively trivial effects except for the urban fringe and housing quality in the outer city. Concerning tax policy parameters, property and gross revenue taxes generate greater effects on the urban fringe,

[6] While most of these findings are robust to parameter changes with reasonable ranges, we would like to note that it is possible that housing quality in outer city may be lower in response to an increase in β or A, or a decrease in ϕ. This case may arise particularly when the urban fringe is very responsive to such parameter shifts.

the population density and housing-related variables, whereas property and land taxes cause larger responses of the land rent schedule (for all locations).

5 Welfare Analysis

In our model economy, all developers earn zero profit in equilibrium and hence their welfare measure is trivial. Moreover, because the household utility function is linear in consumption, the city-wide utilitarian welfare can simply be measured by aggregating the values of households and absentee landlords with equal weights:

$$\Omega \equiv Nu(0) + ALR \tag{35}$$

where the aggregate land rent is defined as:

$$ALR \equiv 2 \int_0^{x_f} R(z)dz.$$

Should the government be more concerned with inequalities between households and absentee landlords, one may consider a more egalitarian social welfare measure using harmonic average of the welfare of households and the welfare of absentee landlords:

$$\Omega^{EGA} \equiv \left(\frac{\frac{1}{y}}{\frac{1}{y} + \frac{1}{ALR}} \right) Nu(0) + \left(1 - \frac{\frac{1}{y}}{\frac{1}{y} + \frac{1}{ALR}} \right) ALR$$

Of course, one may also examine the welfare of an individual household (using $u(0)$) and the welfare of developers (using their after-tax total gross revenue, $(1 - \tau_S) \cdot TGR$).

We next simulate the model to conduct policy and welfare analysis. We are interested in conducting the following "tax incidence" exercises: (i) property tax (τ_H) vs. gross revenue tax (τ_S); (ii) property tax (τ_H) vs. land tax (τ_L); (iii) gross revenue tax (τ_S) vs. land tax (τ_L); and, (iv) gross revenue tax (τ_S) vs. development tax (τ_D). In each of the tax-incidence exercise for a pair of taxes, the local public amenity spending is fixed at G and other tax rates are set at the benchmark value reported in Section 4.2. To solve the optimal mix for each tax pair, we fix G and set other taxes to zero. We have studied various welfare measures as mentioned above, reaching qualitatively similar conclusions. Thus, we focus exclusively on illustrating the best combination of various tax instruments to achieve highest welfare measured by Ω.

The main findings are summarized as follows. First, the land tax is the most preferred tax instrument, reaffirming the idea underlying the Henry George Theorem in the urban economics literature. However, the land tax should not be set at 100%. This is because in our model the land tax is not purely nondistortionary – it will affect the endogenous urban fringe and hence the endogenous housing density schedule. Second, since the development tax generates less distortion (no distortion on capital demand or households' willingness to pay), it is more preferred to the property tax.

Third, it is somewhat surprising that the gross revenue tax is more preferred to the property tax – in the optimal taxation literature under perfect competition, a tax on the supply side usually generates a larger distortion. Our anti-conventional finding is due mainly to household's preference for housing quality and firm's local monopoly power over the housing market. Fourth, although one cannot obtain a clear-cut ranking of gross revenue and development taxes, it is possible to examine the optimal mix of these two taxes under the benchmark values $(\tau_H, \tau_L) = (0.15, 0.125)$. We find that the optimal mix of these taxes is $\tau_S = 6.6\%$ and $\tau_D = 17.5\%$. This together with the tax incidence exercise implies that, by pairwise comparison, the gross revenue tax is more distortionary than the development tax. These findings mentioned above are robust to all welfare measures and a reasonably wide range of parameters ($\pm 20\%$ from their benchmark values).

Finally, we would like to inquire what the best combination of all tax instruments is to achieve highest welfare measured by Ω. Under the assumptions that all tax rates are nonnegative (i.e., no subsidy) and the LPG is entirely financed by these four tax instruments, the welfare-maximizing tax mix is given by: $\tau_H = 0$, $\tau_S = 12.6\%$, $\tau_D = 27.5\%$, and $\tau_L = 20.0\%$. That is, in the absence of income and distortionary factor taxes, a globally optimal tax scheme in the housing market is to eliminate the property tax and to impose a lower gross revenue tax rate than either the development or the land tax.

6 Concluding Remarks

We have developed a competitive spatial equilibrium model with local public amenities to characterize the schedules of housing quality, housing prices, land rent, and population and housing density. We have also evaluated quantitatively the equilibrium effects and welfare implications of an array of housing-related tax policies, including a gross revenue tax, a property tax, a land tax and a development tax. We find that housing quality and housing prices/values are higher in the outer city than in the inner city and that the housing quality and housing prices/values are positively related in response to any preference and technology shifts, consistent with the observations from the American Housing Survey. Moreover, a reduction in the unit commuting cost reduces the number of floors and raises housing quality and prices in the inner city, but generates a reverse outcome in the outer city. Furthermore, a globally optimal tax scheme in the housing market is to eliminate the property tax and to impose a lower gross revenue tax rate than either the development or the land tax.

Along these lines, there are at least three interesting avenues for future research. One is to allow the *service* of public amenities to depend negatively on the distance from the public amenity site (city center). We may now define households' valuation of housing quality to include public amenity services. When households' preferences for public amenties are sufficiently strong and the spatial discount of the public good service is sufficiently large, it is possible to have downward sloping housing price and housing quality schedules, capturing the features in Asian and European cities.[7]

The second is to generalize the model to a dynamic setting such that the durability feature of housing can be incorporated. In this case, one may tie housing quality with the age of the structure. Moreover, a distortionary housing tax may now affect households' investment in housing quality (home improvements) and their intertemporal consumption-saving trade-offs. The last is to take political economy considerations in that the housing tax structure is endogenously determined by voting. It is expected that the voting outcome may be different from the welfare-maximizing policy mix due to the presence of the local public good, local and aggregate externalities and distortionary taxes.

[7] Denoting the service of public amenities as g, one may specify: $g(z) = e^{-\delta z}G$, where $\delta > 0$ measures the spatial discount of the public good service. Then households' valuation of housing quality inclusive of public amenity services is given by, $\beta q(z)^{\theta} + \gamma(\ln G - \delta z)$. Downward sloping housing price and quality gradient can be derived when $\gamma\delta$ is sufficiently high compared to β.

Appendix

Straightforward differentiation under condition N yields the comparative static results as reported in Section 4.1. Because (24) can be rewritten as,

$$n(z) = A^{\frac{1}{\varepsilon}} \left[\left(\frac{1-\tau_S}{1+\tau_H} \right) \frac{\alpha\beta\theta^2}{r} \right]^{\frac{\alpha}{\varepsilon}} \left[q_0 + \frac{t}{\beta(1-\theta)} z \right]^{-\frac{1-\alpha\theta}{\varepsilon\theta}},$$

we obtain:

$$\frac{\partial n(z)}{\partial \beta} \gtreqless 0, \frac{\partial n(z)}{\partial A} \gtreqless 0, \frac{\partial n(z)}{\partial \phi} > 0, \frac{\partial n(z)}{\partial \tau_H} \gtreqless 0, \frac{\partial n(z)}{\partial \tau_S} \gtreqless 0, \frac{\partial n(z)}{\partial \tau_L} > 0, \frac{\partial n(z)}{\partial \tau_D} > 0$$

Further, by rewriting (29) as,

$$R(z) = \frac{(1+\tau_D)\,\phi + (1+\tau_L)\,R_A}{1+\tau_L} \left[\left(1 + D\frac{N}{2} \right)^{\frac{-\varepsilon\theta}{1-(\alpha+\varepsilon)\theta}} + \frac{t}{\beta B(1-\theta)} z \right]^{-\frac{1-(\alpha+\varepsilon)\theta}{\varepsilon\theta}} - \left(\frac{1+\tau_D}{1+\tau_L} \right) \phi$$

we derive:

$$\frac{\partial R(z)}{\partial \beta} > 0, \frac{\partial R(z)}{\partial A} > 0, \frac{\partial R(z)}{\partial \phi} \gtreqless 0, \frac{\partial R(z)}{\partial \tau_H} < 0, \frac{\partial R(z)}{\partial \tau_S} < 0, \frac{\partial R(z)}{\partial \tau_L} \gtreqless 0, \frac{\partial R(z)}{\partial \tau_D} \gtreqless 0$$

Finally, we can show:

$$\frac{\partial ALR}{\partial \beta} > 0, \frac{\partial ALR}{\partial A} > 0, \frac{\partial ALR}{\partial \phi} \gtreqless 0, \frac{\partial ALR}{\partial \tau_H} < 0, \frac{\partial ALR}{\partial \tau_S} < 0, \frac{\partial ALR}{\partial \tau_L} \gtreqless 0, \frac{\partial ALR}{\partial \tau_D} \gtreqless 0$$

$$\frac{\partial \Omega}{\partial \beta} > 0, \frac{\partial \Omega}{\partial A} > 0, \frac{\partial \Omega}{\partial \phi} \gtreqless 0, \frac{\partial \Omega}{\partial \tau_H} < 0, \frac{\partial \Omega}{\partial \tau_S} < 0, \frac{\partial \Omega}{\partial \tau_L} \gtreqless 0, \frac{\partial \Omega}{\partial \tau_D} \gtreqless 0$$

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Table 1: Comparative statics with respective to preference and technology parameters

	x^f	$u(0)$	Ω	$\frac{ACC}{Ny}$	$\frac{ALR}{Ny}$	$\frac{TGR}{Ny}$	$\frac{TDF}{TGR}$	$\frac{G}{Ny}$
Benchmark	14.70	2.53	2556	0.0450	0.0251	0.0741	0.119	0.0205
$\beta \cdot 0.8$	6.02	2.08	2085	0.0184	0.0103	0.0304	0.119	0.0084
	(-59.05)	(-17.79)	(-18.43)	(-59.11)	(-58.96)	(-58.97)	(0)	(-59.02)
$\beta \cdot 0.9$	9.65	2.31	2331	0.0295	0.0164	0.0486	0.119	0.0135
	(-34.35)	(-8.70)	(-8.80)	(-34.44)	(-34.66)	(-34.41)	(0)	(-34.15)
$\beta \cdot 1.1$	21.53	2.73	2764	0.0659	0.0367	0.1086	0.119	0.03
	(46.43)	(7.91)	(8.14)	(46.44)	(46.22)	(46.56)	(0)	(46.34)
$\beta \cdot 1.2$	30.49	2.91	2960	0.0933	0.052	0.1537	0.119	0.0426
	(107.39)	(15.02)	(15.81)	(107.33)	(107.17)	(107.42)	(0)	(107.80)
$A \cdot 0.8$	8.61	2.25	2272	0.0263	0.0147	0.0434	0.119	0.012
	(-41.43)	(-11.07)	(-11.11)	(-41.56)	(-41.43)	(-41.43)	(0)	(-41.46)
$A \cdot 0.9$	11.42	2.40	2421	0.0349	0.0195	0.0576	0.119	0.0159
	(-22.31)	(-5.14)	(-5.28)	(-22.44)	(-22.31)	(-22.27)	(0)	(-22.44)
$A \cdot 1.1$	18.48	2.65	2679	0.0566	0.0315	0.0932	0.119	0.0258
	(25.71)	(4.74)	(4.81)	(25.78)	(25.50)	(25.78)	(0)	(25.85)
$A \cdot 1.2$	22.77	2.76	2794	0.0697	0.0388	0.1148	0.119	0.0318
	(54.90)	(9.09)	(9.31)	(54.89)	(54.58)	(54.93)	(0)	(55.12)
$\phi \cdot 0.8$	17.89	2.56	2591	0.0487	0.0275	0.0791	0.1086	0.0219
	(21.70)	(1.19)	(1.37)	(8.22)	(9.56)	(6.75)	(-8.74)	(6.83)
$\phi \cdot 0.9$	16.16	2.55	2573	0.0468	0.0262	0.0765	0.114	0.0212
	(9.93)	(0.79)	(0.67)	(4.00)	(4.38)	(3.24)	(-4.20)	(3.41)
$\phi \cdot 1.1$	13.47	2.52	2539	0.0434	0.024	0.072	0.1235	0.0199
	(-8.37)	(-0.40)	(-0.67)	(-3.56)	(-4.38)	(-2.83)	(3.78)	(-2.93)
$\phi \cdot 1.2$	12.41	2.50	2524	0.0419	0.0231	0.07	0.1276	0.0194
	(-15.58)	(-1.19)	(-1.25)	(-6.89)	(-7.97)	(-5.53)	(7.23)	(-5.37)
$t \cdot 0.8$	18.27	2.62	2650	0.0524	0.0294	0.0883	0.1241	0.0244
	(24.29)	(3.56)	(3.68)	(16.44)	(17.13)	(19.16)	(4.29)	(19.02)
$t \cdot 0.9$	16.29	2.57	2600	0.0484	0.027	0.0806	0.1214	0.0223
	(10.82)	(1.58)	(1.72)	(7.56)	(7.57)	(8.77)	(2.02)	(8.78)
$t \cdot 1.1$	13.39	2.49	2515	0.0421	0.0234	0.0687	0.1169	0.019
	(-8.91)	(-1.58)	(-1.60)	(-6.44)	(-6.77)	(-7.29)	(-1.76)	(-7.32)
$t \cdot 1.2$	12.30	2.46	2478	0.0396	0.0219	0.0641	0.1151	0.0178
	(-16.33)	(-2.77)	(-3.05)	(-12.00)	(-12.75)	(-13.50)	(-3.28)	(-13.17)

Note: The percentage changes of the responses are reported in parentheses (in %).

Table 1 (continued)

	$n(0)$	$n(x^f)$	$q(0)$	$q(x^f)$	$h^s(0)$	$h^s(x^f)$	$v(0)$	$v(x^f)$	$R(0)$
Benchmark	1460	1.10	0.0414	28.73	60.60	31.50	0.0154	0.7825	9.93
$\beta \cdot 0.8$	3570	2.68	0.0136	9.41	48.48	25.20	0.0063	0.3205	9.93
	(144.0)	(144.0)	(-67.3)	(-67.3)	(-20.0)	(-20.0)	(-59.1)	(-59.1)	(0)
$\beta \cdot 0.9$	2230	1.67	0.0245	16.96	54.54	28.35	0.0101	0.5134	9.93
	(52.0)	(52.0)	(-40.9)	(-40.9)	(-10.0)	(-10.0)	(-34.4)	(-34.4)	(0)
$\beta \cdot 1.1$	999	0.75	0.0667	46.27	66.66	34.65	0.0226	1.1457	9.93
	(-31.7)	(-31.7)	(61.1)	(61.1)	(10.0)	(10.0)	(46.4)	(46.4)	(0)
$\beta \cdot 1.2$	706	0.53	0.1031	71.49	72.72	37.80	0.032	1.6226	9.93
	(-51.7)	(-51.7)	(148.9)	(148.9)	(20.0)	(20.0)	(107.6)	(107.6)	(0)
$A \cdot 0.8$	2500	1.87	0.017	11.77	42.40	22.04	0.009	0.458	9.93
	(71.0)	(71.0)	(-58.9)	(-58.9)	(-30.0)	(-30.0)	(-41.6)	(-41.6)	(0)
$A \cdot 0.9$	1880	1.41	0.0272	18.85	51.20	26.61	0.012	0.6077	9.93
	(28.5)	(28.5)	(-34.3)	(-34.3)	(-15.5)	(-15.5)	(-22.2)	(-22.2)	(0)
$A \cdot 1.1$	1160	0.87	0.0606	42.06	70.58	36.69	0.0194	0.9836	9.93
	(-20.8)	(-20.8)	(46.4)	(46.4)	(16.5)	(16.5)	(25.9)	(25.9)	(0)
$A \cdot 1.2$	945	0.71	0.0859	59.57	81.12	42.17	0.0239	1.212	9.93
	(-35.4)	(-35.4)	(107.4)	(107.4)	(33.9)	(33.9)	(55.5)	(54.5)	(0)
$\phi \cdot 0.8$	1440	0.77	0.0419	39.62	60.53	30.51	0.0156	0.9489	9.93
	(-1.4)	(-30.0)	(1.21)	(37.9)	(-0.12)	(-3.14)	(1.30)	(21.3)	(0)
$\phi \cdot 0.9$	1450	0.93	0.0417	33.52	60.56	31.02	0.0155	0.8584	9.93
	(-0.68)	(-15.5)	(0.72)	(16.7)	(-0.07)	(-1.52)	(0.65)	(9.70)	(0)
$\phi \cdot 1.1$	1470	1.28	0.0412	24.90	60.63	31.96	0.0154	0.7181	9.93
	(0.68)	(16.4)	(-0.48)	(-13.3)	(0.05)	(1.46)	(0.00)	(-8.23)	(0)
$\phi \cdot 1.2$	1480	1.49	0.041	21.78	60.66	32.39	0.0153	0.6627	9.93
	(1.37)	(35.5)	(-0.97)	(-24.2)	(0.10)	(2.83)	(-0.65)	(-15.3)	(0)
$t \cdot 0.8$	914	1.1	0.0635	28.73	58.06	31.5	0.02	0.7825	7.96
	(-37.4)	(0)	(53.4)	(0)	(-4.19)	(0)	(29.9)	(0)	(-19.8)
$t \cdot 0.9$	1170	1.1	0.0507	28.73	59.38	31.5	0.0174	0.7825	8.95
	(-19.9)	(0)	(22.5)	(0)	(-2.01)	(0)	(13.0)	(0)	(-9.87)
$t \cdot 1.1$	1790	1.1	0.0345	28.73	61.72	31.5	0.0138	0.7825	10.91
	(22.6)	(0)	(-16.7)	(0)	(1.85)	(0)	(-10.4)	(0)	(9.87)
$t \cdot 1.2$	2150	1.1	0.0291	28.73	62.77	31.5	0.0125	0.7825	11.90
	(47.3)	(0)	(-29.7)	(0)	(3.58)	(0)	(-18.8)	(0)	(19.8)

Table 2: Camparative statics with respect to tax policy parameters

	x^f	$u(0)$	Ω	$\frac{ACC}{Ny}$	$\frac{ALR}{Ny}$	$\frac{TGR}{Ny}$	$\frac{TDF}{TGR}$	$\frac{G}{Ny}$
Benchmark	14.70	2.53	2556	0.0450	0.0251	0.0741	0.119	0.0205
$\tau_H \cdot 0.8$	15.92	2.52	2545	0.0478	0.0273	0.0807	0.1184	0.0199
	(8.30)	(-0.40)	(-0.43)	(6.22)	(8.76)	(8.91)	(-0.50)	(-2.93)
$\tau_H \cdot 0.9$	15.30	2.52	2551	0.0464	0.0262	0.0773	0.1187	0.0202
	(4.08)	(-0.40)	(-0.20)	(3.11)	(4.38)	(4.32)	(-0.25)	(-1.46)
$\tau_H \cdot 1.1$	14.14	2.54	2559	0.0437	0.024	0.0711	0.1193	0.0208
	(-3.81)	(0.40)	(0.12)	(-2.89)	(-4.38)	(-4.05)	(0.25)	(1.46)
$\tau_H \cdot 1.2$	13.60	2.54	2562	0.0424	0.023	0.0683	0.1195	0.0209
	(-7.48)	(0.40)	(0.23)	(-5.78)	(-8.37)	(-7.83)	(0.42)	(1.95)
$\tau_S \cdot 0.8$	15.43	2.53	2552	0.0467	0.0264	0.0768	0.1205	0.0202
	(4.97)	(0.00)	(-0.16)	(3.78)	(5.18)	(3.64)	(1.26)	(-1.46)
$\tau_S \cdot 0.9$	15.07	2.53	2556	0.0458	0.0257	0.0755	0.1198	0.0204
	(2.52)	(0.00)	(0)	(1.78)	(2.39)	(1.89)	(0.67)	(-0.49)
$\tau_S \cdot 1.1$	14.34	2.54	2561	0.0442	0.0244	0.0728	0.1182	0.0207
	(-2.45)	(0.40)	(0.20)	(-1.78)	(-2.79)	(-1.75)	(-0.67)	(0.98)
$\tau_S \cdot 1.2$	13.99	2.54	2563	0.0433	0.0238	0.0715	0.1174	0.0208
	(-4.83)	(0.40)	(0.27)	(-3.78)	(-5.18)	(-3.51)	(-1.34)	(1.46)
$\tau_L \cdot 0.8$	14.81	2.52	2546	0.0451	0.0256	0.0743	0.1195	0.02
	(0.75)	(-0.40)	(-0.39)	(0.22)	(1.99)	(0.27)	(0.42)	(-2.44)
$\tau_L \cdot 0.9$	14.75	2.53	2552	0.0451	0.0254	0.0742	0.1193	0.0203
	(0.34)	(0.00)	(-0.16)	(0.22)	(1.20)	(0.13)	(0.25)	(-0.98)
$\tau_L \cdot 1.1$	14.65	2.54	2564	0.0449	0.0248	0.0741	0.1187	0.0208
	(-0.34)	(0.40)	(0.31)	(-0.22)	(-1.20)	(0.00)	(-0.25)	(1.46)
$\tau_L \cdot 1.2$	14.60	2.55	2570	0.0449	0.0245	0.074	0.1184	0.021
	(-0.68)	(0.79)	(0.55)	(-0.22)	(-2.39)	(-0.13)	(-0.50)	(2.44)
$\tau_D \cdot 0.8$	14.90	2.53	2558	0.0453	0.0252	0.0745	0.1201	0.0205
	(1.36)	(0.00)	(0.08)	(0.67)	(0.40)	(0.54)	(0.92)	(0)
$\tau_D \cdot 0.9$	14.80	2.53	2558	0.0451	0.0251	0.0743	0.1195	0.0205
	(0.68)	(0.00)	(0.08)	(0.22)	(0.00)	(0.27)	(0.42)	(0)
$\tau_D \cdot 1.1$	14.60	2.53	2559	0.0449	0.025	0.074	0.1184	0.0205
	(-0.68)	(0.00)	(0.12)	(-0.22)	(-0.40)	(-0.13)	(-0.50)	(0)
$\tau_D \cdot 1.2$	14.51	2.53	2559	0.0448	0.0249	0.0738	0.1179	0.0206
	(-1.29)	(0.00)	(0.12)	(-0.44)	(-0.80)	(-0.40)	(-0.92)	(0.49)

Table 2 (continued)

	$n(0)$	$n(x^f)$	$q(0)$	$q(x^f)$	$h^s(0)$	$h^s(x^f)$	$v(0)$	$v(x^f)$	$R(0)$
Benchmark	1460	1.10	0.0414	28.73	60.60	31.50	0.0154	0.7825	9.93
$\tau_H \cdot 0.8$	1390	0.99	0.0449	32.79	62.54	32.35	0.0167	0.8698	10.19
	(-4.79)	(-10.00)	(8.45)	(14.13)	(3.20)	(2.70)	(8.44)	(11.16)	(2.62)
$\tau_H \cdot 0.9$	1430	1.04	0.0431	30.68	61.55	31.92	0.016	0.8247	10.06
	(-2.05)	(-5.45)	(4.11)	(6.79)	(1.57)	(1.33)	(3.90)	(5.39)	(1.31)
$\tau_H \cdot 1.1$	1500	1.15	0.0398	26.93	59.67	31.10	0.0149	0.743	9.80
	(2.74)	(4.55)	(-3.86)	(-6.27)	(-1.53)	(-1.27)	(-3.25)	(-5.05)	(-1.31)
$\tau_H \cdot 1.2$	1540	1.22	0.0383	25.26	58.76	30.70	0.0144	0.7059	9.68
	(5.48)	(10.91)	(-7.49)	(-12.08)	(-3.04)	(-2.54)	(-6.49)	(-9.79)	(-2.52)
$\tau_S \cdot 0.8$	1420	1.03	0.0435	31.14	61.77	32.01	0.0159	0.8212	10.09
	(-2.74)	(-6.36)	(5.07)	(8.39)	(1.93)	(1.62)	(3.25)	(4.95)	(1.61)
$\tau_S \cdot 0.9$	1440	1.06	0.0425	29.91	61.18	31.76	0.0157	0.8017	10.01
	(-1.37)	(-3.64)	(2.66)	(4.11)	(0.96)	(0.83)	(1.95)	(2.45)	(0.81)
$\tau_S \cdot 1.1$	1490	1.13	0.0404	27.58	60.01	31.25	0.0152	0.7636	9.85
	(2.05)	(2.73)	(-2.42)	(-4.00)	(-0.97)	(-0.79)	(-1.30)	(-2.42)	(-0.81)
$\tau_S \cdot 1.2$	1451	1.17	0.0394	26.47	59.43	30.99	0.015	0.745	9.77
	(-0.62)	(6.36)	(-4.83)	(-7.87)	(-1.93)	(-1.62)	(-2.60)	(-4.79)	(-1.61)
$\tau_L \cdot 0.8$	1460	1.08	0.0414	29.06	60.59	31.47	0.0154	0.7879	10.15
	(0)	(-1.82)	(0)	(1.15)	(-0.02)	(-0.10)	(0)	(0.69)	(2.22)
$\tau_L \cdot 0.9$	1460	1.09	0.0414	28.90	60.59	31.48	0.0154	0.7852	10.04
	(0)	(-0.91)	(0)	(0.59)	(-0.02)	(-0.06)	(0)	(0.35)	(1.11)
$\tau_L \cdot 1.1$	1460	1.1	0.0414	28.57	60.60	31.52	0.0154	0.7798	9.82
	(0)	(0.00)	(0)	(-0.56)	(0.00)	(0.06)	(0)	(-0.35)	(-1.11)
$\tau_L \cdot 1.2$	1460	1.11	0.0414	28.40	60.60	31.54	0.0154	0.7772	9.72
	(0)	(0.91)	(0)	(-1.15)	(0.00)	(0.13)	(0)	(-0.68)	(-2.11)
$\tau_D \cdot 0.8$	1460	1.07	0.0415	29.37	60.59	31.43	0.0155	0.793	9.93
	(0)	(-2.73)	(0.24)	(2.23)	(-0.02)	(-0.22)	(0.65)	(1.34)	(0)
$\tau_D \cdot 0.9$	1460	1.08	0.0414	29.05	60.59	31.47	0.0154	0.7877	9.93
	(0)	(-1.82)	(0)	(1.11)	(-0.02)	(-0.10)	(0)	(0.66)	(0)
$\tau_D \cdot 1.1$	1460	1.11	0.0414	28.42	60.60	31.54	0.0154	0.7774	9.93
	(0)	(0.91)	(0)	(-1.08)	(0.00)	(0.13)	(0)	(-0.65)	(0)
$\tau_D \cdot 1.2$	1460	1.12	0.0414	28.11	60.60	31.57	0.0154	0.7723	9.93
	(0)	(1.82)	(0)	(-2.16)	(0.00)	(0.22)	(0)	(-1.30)	(0)

Figure 1. Equilibrium determination and effects of a reduction in ϕ or an increase in A

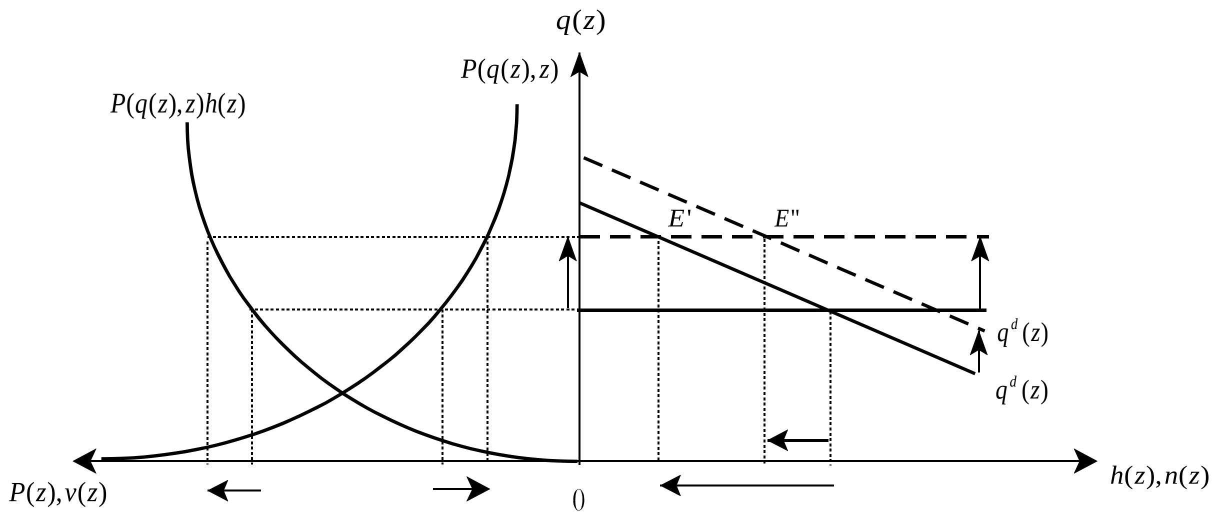

Figure 2. Effects of an increase in β or A, or a decrease in ϕ

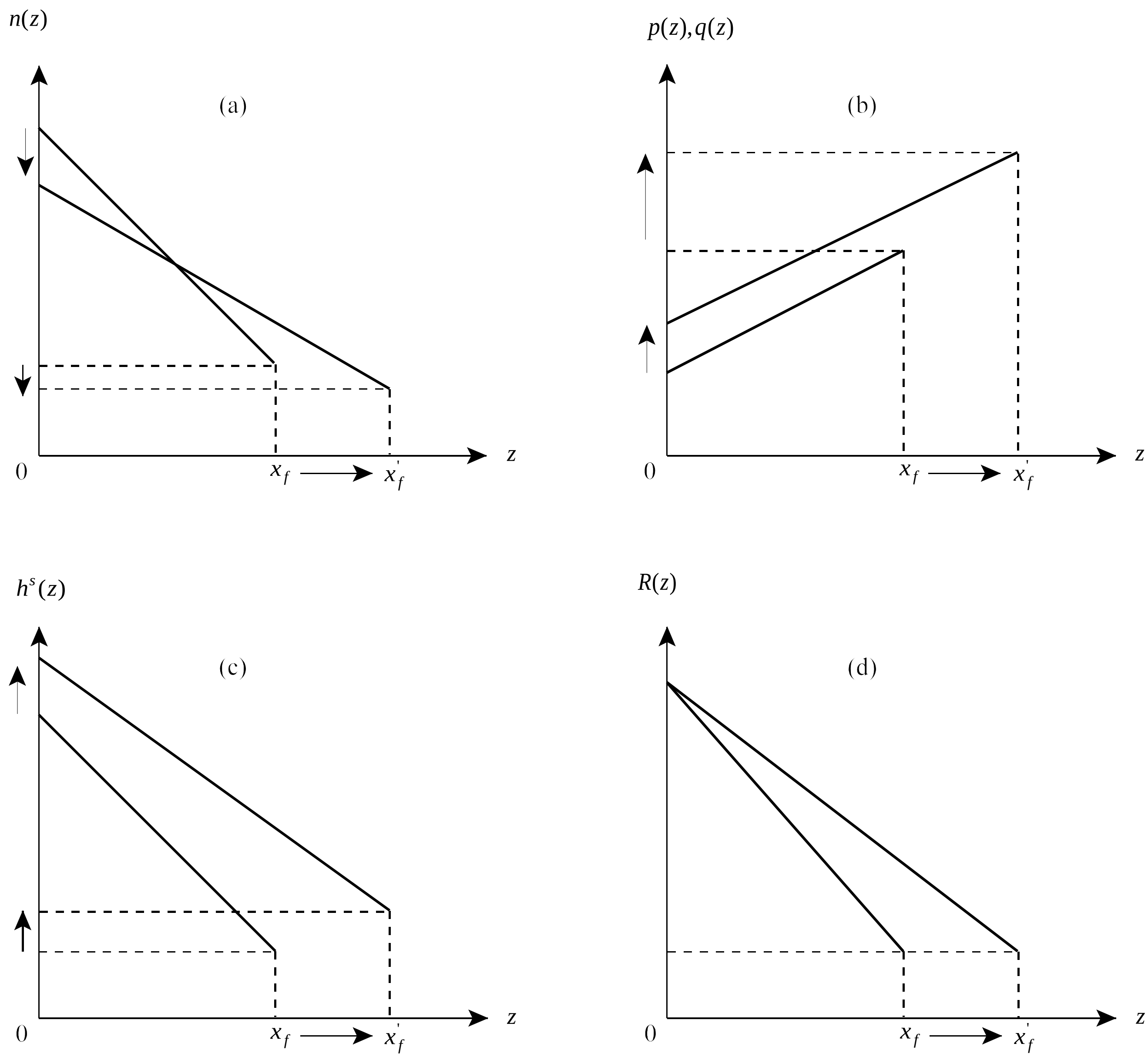

Figure 3. Effects of a reduction in *t*